Franck-Hertz-Experiment

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Abstract. The Franck-Hertz-Experiment, first performed by J. Franck and G.Hertz in 1913, proves that atoms can absorb energy only in discrete quanta, even when the excitation is due to collisions. Up to that point, this had only been proven for optical excitation.

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Contents

Sources

H.Haken, H.C.Wolf: Atom- und Quantenphysik, Springer Verlag

P.A.Tipler: Physik, Spektrum Lehrbuch

as well as protocolls of other students and the internet.

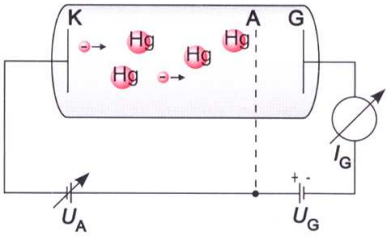
Layout (.sty-file für LaTeX 2_ε) by Norbert Stuhrmann, 2003.

1. Theory

1.1. Franck-Hertz-Tube

The Franck-Hertz-Tube itself is a vacuum tube containing a drop of mercury. There are three electrodes prepared inside, the thermionic cathode, a grid-like anode and an electrode at the back which finally collects the electrons. The tube is located inside an oven and due to the higher temperature some of the mercury dissolves.

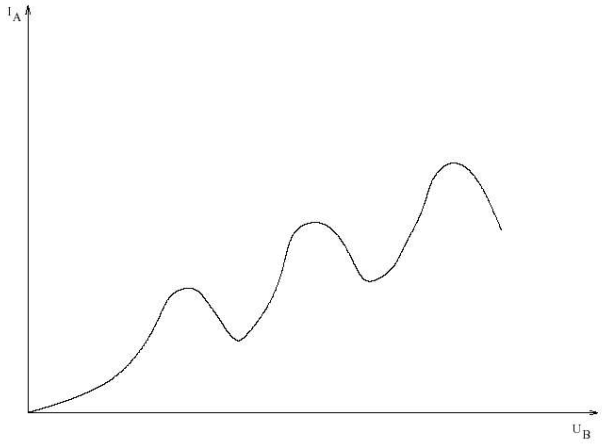
The high acceleration potential between the cathode and the anode results in an acceleration of the electrons. A weak retarding field towards the back electrode only allows electrons with a certain minimum velocity to reach the back.



At low anode voltages, the electrons undergo only *elastic* collisions with the mercury atoms (ref. chapter 1.3) barely losing any energy. But when the anode voltage is high enough, the electrons gain enough energy to excite a mercury atom in an inelastic collision. Those electrons will no longer have enough energy to pass the retarding field, so the output current decreases. Assuming that an electron has enough energy, it can excite several mercury atoms.

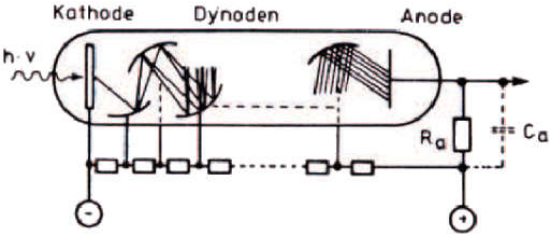
By continuisly increasing the accelleration voltage, the drop in the current is seen at each multiple of the excitation voltage, simply beause there, most of the electrons where able to pass their entire kinetic energy towards the atoms, leaving them to slow to reach the back electrode. Due to the maxwell-disributed velocities of the electrons, the drop will never go all the way to 0A.

The excitation energy of mercury can be calculated from the distance of the minima/maxima of the resulting graph. It will look something like this:



1.2. Photomultiplier

Photomultipliers are photon detectors, which make use of the photoelectric effect. They are constructed out of a vacuum tube with a photocathode at its front side. An incoming photon knocks an electron from the cathode. The electron is directed by an accelerating field to several dynodes which are arranged one after the other.



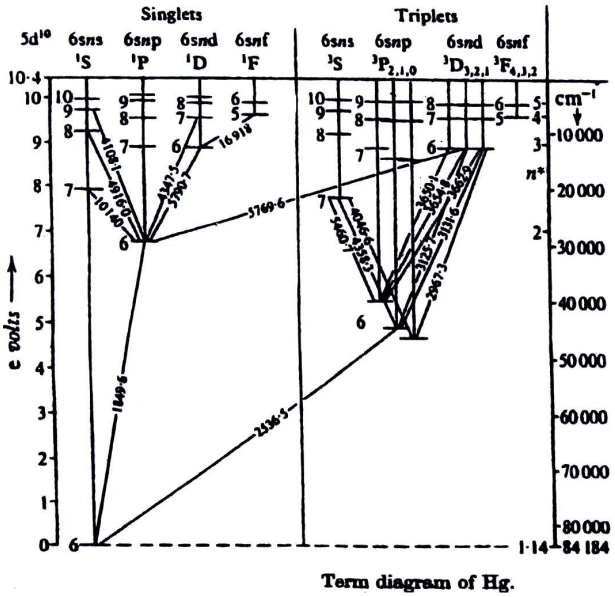
This is accomplished by using a voltage divider, so that the dynodes are all set at different, increasing potentials. Since the electrons have gained kinetic energy in the field, each electron produces several secondary electrons at each dynode. This multiplies the electrons, the amplified current is collected at an anode, giving the output signal.

1.3. The mercury atom

The data gained by this experiemnt is mostly affected by properties of the mercury atom. Mercury is a metal with atomic number $Z = 80$. It is liquid at room temperatures. It's atomic mass has been measured as

$$m_{Hg} = 200.59u = 3.33 \cdot 10^{-22} g$$

This clearly exceeds the electron mass by orders of magnitude. Therefore, electrons elasticly colliding with mercury atoms keep most of their kinetic energy. Only if the electron energy is high enough to excite the mercury atom, an inelastic collision can occure. As seen in the following diagram, the two outer s-electrons of mercury have three neighbourghing p-states as their lowest energy states.



The energies of the possible transitions are 4.67V, 4.89V, and 5.46V

Since the pressure in the vacuum tube is very high, the collision probability is very high as well, leading to a short free path lenght. As a result only these three lowest states are of our interest. More precisely, these three states can be excited by the electrons, but only the 6^3P_1 can perform an optical transition back to the ground state. This is due to the selection rules which follow from quantum mechanical calculations (ref. Haken/Wolf). The remaining two states can only drop back to the ground state by colliding with other mercury atoms.

The free path lenght of the electrons can be calculated as follows:

$$\Lambda = \frac{1}{\nu\sigma}$$

The particle density $\nu = n \cdot \frac{Na}{V}$ can be derived from the ideal-gas-equation:

$$p = \nu \cdot k_B \cdot T$$

The cross section σ for a 5 eV - electron is given by the graph in the script:

$$\sigma \approx 3.5 \cdot \pi a_0^2 \approx 0.031 \text{ nm}^2$$

The pressure follows from the formula in the script, which itself can be derived by integrating the Clausius-Clapeyron-equation:

$$\log_{10} p = 10.55 - \frac{3333}{T} - 0.85 \log_{10} T$$

Plugging all this together and assuming a mean temperatur of 160°C, we obtain

$$\Lambda = \frac{k_B \cdot T}{p \cdot \sigma} = \frac{1.38 \cdot 10^{-31} \text{ JK}^{-1} \cdot 433 \text{ K}}{548 \text{ Pa} \cdot 3.1 \cdot 10^{-20} \text{ m}^2}$$

and thus

$$\Lambda \approx 3.55 \cdot 10^{-4} \text{ m}$$

A similar calculation delivers the mean collision time of two mercury atoms, it is given by:

$$\tau = \frac{\Lambda}{v}$$

like

$$\sigma = \pi \cdot (r_{Hg} + r_{Hg})^2 = \pi \cdot (0.44 \text{ nm})^2$$

$$= 6.08 \cdot 10^{-19} \text{ m}^2$$

and thus

$$\Lambda = \frac{1.38 \cdot 10^{-31}\ \mathrm{JK}^{-1} \cdot 433\ \mathrm{K}}{548\ \mathrm{Pa} \cdot 6.08 \cdot 10^{-19}\ \mathrm{m}^2} = 1.79 \cdot 10^{-5}\ \mathrm{m}$$

The velocities of the mercury atoms are maxwell-disributed, and therefore

$$v = \sqrt{\frac{8RT}{\pi M_{Hg}}} = \sqrt{\frac{8 \cdot 8.31\ \mathrm{J\ (mol\ K)^{-1}} \cdot 433\ \mathrm{K}}{\pi \cdot 0.2006\ \mathrm{kg\ mol^{-1}}}}$$

$$\approx 214\ \frac{\mathrm{m}}{\mathrm{s}}$$

The last step delivers the desired mean collision time:

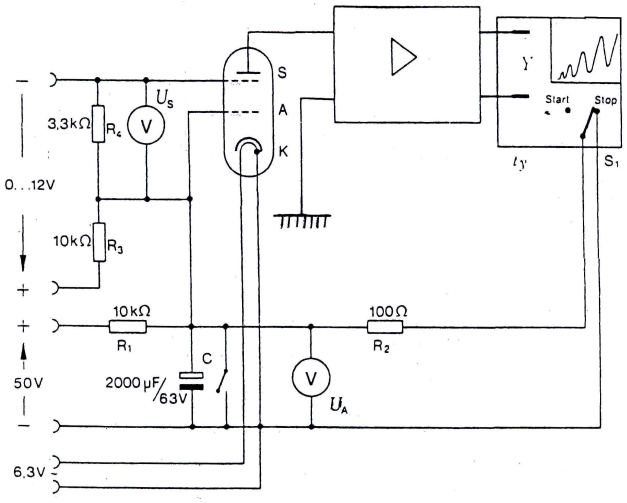
$$\tau = \frac{1.79 \cdot 10^{-5}\ \mathrm{m}}{214\ \frac{\mathrm{m}}{\mathrm{s}}} \approx 8.39 \cdot 10^{-8}\ \mathrm{s}$$

Comparing this to the lifetime of e.g. the 6^3P_1 state ($1.17 \cdot 10^{-7}$ s), we find that an excited mercury atom will perform several collisions with other atoms, before emitting their photon. Thus, we expect to find photons of different wavelengths arriving at the photomultiplier.

2. Experimental setup

The following sketch shows the principal features of the experimental setup. The current from the back electrode is first amplified and then used for the y-input for the plotter. The x-input of the plotter is connected with the grid voltage of the anode.

When starting a measurement, the capacitor is loaded, constantly increasing the grid voltage. The switch serves to shortcut the capacitor and quickly decrease the grid voltage. In the sketch, the back voltage is named U_S and the grid voltage U_A.



Alternatively, a photomultipier can be put in front of a UV-transparent window in the Franck-Hertz-tube and be connected with the amplifier. Then, the plotter draws the photocurrent curve. To get more usefull information out of the plotter, we can add a differentiating circuit just before the y-input of the plotter. The plotter then draws the first derivative of the actual graph.

There is one thing to be mentioned about the tube, at low temperatures (low collision-probability) electrons can reach much higher energies and therefore ionize the mercury gas, which eventually leads to a light flash within the tube causing damage. Therefore it was important to switch on the cathode only after heating it to at least 140°C.

3. Execution of the experiment and analysis of the data

Before we started to collect data, we tried out different settings at the plotter and the amplefier, to make sure, that all graphs fit on one paper, without changing the resolutions. This is necessary to be able to compare the graphs.

One graph however still went „wrong“, we think that the temperature was slightly different as when we tested the resolution. This slight change in temperatur caused the increase in the anode-current.

3.1. Description of the experiment

To understand the influence of the different parameters, we executed the experiment with the following setups:

- $U_{B(max)} = 41\text{V}$
- $U_G = 1\text{V}$ at T = (140, 150, 165 and 175°C)
- T = 174°C at U_G = (1V and 2V)

Each measurement resulted in a graph written by an x-y-writer. The x-input signal was deliverd by the linear increasing U_G (max 41V) and the y-input was taken from the anode current in the tube.

The x-axis must be scaled by the maximum acceleration voltage, 41V $\hat{=}$ 32.6 cm.

From the graph of each measurement, we get the mean excitation energy. By using

$$\lambda = \frac{h \cdot c}{E}$$

we geht the corresponding wavelength. The problem we see with this, the resulting excitation energy is the mean of *all* transitions including the two transitions that can loose their energy only by collisions and not by emitting photons.

The wavelenght expected from the $4,89$ eV transition

$$\lambda = 253,6 \text{ nm}$$

3.2. Constant $U_G = 1V$ and variable temparatures (sheet A)

The four graphs correspond to the following temperatures:

- 140°C (red curve)
- 150°C (blue curve)
- 165°C (green curve)
- 175°C (black curve)

Table 1: Variation of temperature

T [°C]	d [cm]	d [V]	∅ d [V]	λ [nm]
140	3.8	4.78	4.89	254
	3.85	4.84	± 0.13	± 6.74
	4.0	5.03		
	3.9	4.90		
150	3.8	4.78	4.87	254
	3.8	4.78	± 0.13	± 6.79
	3.9	4.90		
	3.9	4.90		
	3.9	4.90		
	3.95	4.97		
165	3.85	4.84	4.81	258
	3.85	4.84	± 0.13	± 6.97
	3.8	4.78		
	3.8	4.78		
175	3.65	4.59	4,62	268
	3.65	4.59	± 0.13	± 7.55
	3.7	4.65		
	3.7	4.65		

The error for estimating the distance between maxima is roughly Δ d = 1 mm (Δ U = 0.13 V), the results are listed in table 1.

All calculated values lie nicely around the expected excitation voltages of the p-states (4.67V, 4.89V, and 5.46V), although the highest p-state is apparently barely ever occupied. This makes sense, because in order to excite an atom to the 6^3P_2 state, an electron has to gain 0.79eV of energy more than would be sufficient to push the mercury to the lowest excited state. This means the electron has to „survive“ inside the gas without colliding into an atom long enough to reach 5.46eV of energy, and this is very unlikely.

We can also observe that the distance between two peaks decreases when we raise the temperature. This is also to be expected, because a higher temperature also induces a higher pressure. The higher the pressure, the smaller the mean free path of the electrons, and the higher the collision probability. So with increasing temperature, we observe a shift of the possible mercury transitions towards the energetically lowest ones (from 6^3P_1 to nearly only 6^3P_0 transitions).

3.3. Variable U_G and constant temparature T=174°C (sheet B)

The two graphs correspond to the following retarding potentials:

- 1V (green curve)
- 2V (red curve)

The error we make by estimating the distance between maxima is the same as above, so Δ U = 0.13 V, the results are listed in table 2.

Table 2: Variation of U_G

U_G [V]	d [cm]	d [V]	∅ d [V]	λ [nm]
1V	3.7	4.65	4.65	266
	3.65	4.59	± 0.13	± 7.44
	3.70	4.65		
	3.75	4.72		
2V	3.65	4.59	4.67	266
	3.7	4.65	± 0.13	± 7.39
	3.7	4.65		
	3.8	4.78		

Again we see a neat fit of our results with what we would expect. Although the higher retarding potential compresses our graph, the position of the peaks don't change.

3.4. Measuring the photo current (sheet C)

This last measurement was by far the sloppiest. The graphs show no clear peaks or turning points, at least we cannot pin them down precisely. Therefore the measurment error is huge (Δ U = 0.63 V), and we should not pay too much attention to the absolute values in voltage.

However, the measurement is very useful for qualitative explanation of the processes inside the tube. What we do see is that at first no photons are emitted. Then, we see a continuous increase of the photon emission and an increase of its slope roughly every 5V. Before the experiment we expected the curve of the photo current to look like the measured derivative. We thought that, once the electrons had enough energy, they *all* excited one mercury atom, and either fell back to the grid or reach the back electrode. In this model, the photo current should remain constant between two peaks.

Table 3: Variation of temperature (photo current)

T [°C]	d [cm]	d [V]	∅ d [V]	λ [nm]
140	3.75	4.72	4.72	263
	3.75	4.72	± 0.63	± 35.1
150	3.7	4.65	4.73	262
	3.9	4.90	± 0.63	± 34.9
	3.85	4.84		
	3.6	4.53		
165	3.6	4.53	4.68	265
	3.9	4.90	± 0.63	± 35.6
	3.8	4.78		
	3.6	4.53		
	(4.50)	(5.66)		
175	3.9	4.90	4,78	260
	3.6	4.53	± 0.63	± 34.2
	3.9	4.90		

Values in brackets were disregarded.

What we see now, is the continuous increase of the photon emission. We explain this by thinking of the gas and the electron cloud as rather chaotic. This means that even with an acceleration potential much higher than 5V, only a small fracture of the electrons actually excite the mercury, most of the other electrons loose their energy by elastic collisions and never get beyond the grid. This is why an increase of the acceleration voltage from lets say 6V to 8V results in an increase of the photo current.

In addition to this we observe a second process, the increase of the slope from peak to peak (of the anode current). This is due to the ability of the electrons to excite the atoms multiple times. So what we actually see is the overlap of these two processes.

In closing, it should be mentioned that the biggest errors in these measurements occure when we read out the graphs by hand (or eye). These errors exceed the remaining (e.g. the errors in the measurment instruments) by far, and so it is sufficient to only take these into consideration.